REVIEWS

Yabuuti Kiyosi, *Une histoire des mathématiques chinoises* (*A History of Chinese Mathematics*). Translated by Catherine Jami and Kaoru Baba. Paris: Belin-Pour la science, 2000. 192 pp.

Alexeï Volkov

[Alexeï Volkov is a course lecturer in McGill University, Montreal, Canada. His research is concerned with the history of mathematics in China and Vietnam.]

This book, written by one of the greatest historians of Asian science, the late Professor Yabuuchi Kiyoshi (Yabuuti Kiyosi) 藪内 清 (1906-2000), and skilfully translated by experts both in the history of mathematics and in Japanese, Catherine Jami and Kaoru Baba, was destined to be a success. Despite its relatively small format and the fact that it was originally written as a book intended to popularise the subject, it provides the reader with a wealth of data, and is arguably one of the most complete and skilfully written accounts of the history of Chinese mathematics available in Western languages, featuring equally well the social and cognitive aspects of the discipline.

The contents of the book can be briefly summarised as follows. It opens with a foreword by the translators introducing the author,[1] whose short Introduction is followed by eight chapters; at the end of the book the reader will find a geographical map, a chronological table, and several indexes. The book is organised chronologically. Chapter 1, "The mathematics of antiquity," is devoted to the most ancient period of the history of the discipline, and treats mainly the most ancient numerical notation and counting devices. Chapter 2, "The universe of the Nine Chapters on Mathematical Art," is devoted exclusively to the *Jiuzhang suanshu* 九 章 算 術 (*Computational Procedures of Nine Categories*, widely known nowadays as *Nine Chapters on Mathematical Procedures*); the period under scrutiny approximately corresponds to the Han dynasty (206 BC – AD 220). Chapter 3, "From Six Dynasties to the Tang," is devoted to the computation of the value of π by Liu Hui 劉 徽 (fl. ca. 263), and infinitesimal methods of Zu Chongzhi 祖 沖 之 (429-500) and his son Zu Gengzhi 祖 　 之 (active 504-525).[2] This chapter also contains a general presentation of the

[1] Sadly, Professor Yabuuchi passed away soon after the French translation was published. However, according to a private communication from one of the translators, Catherine Jami, he had seen it printed.

[2] One of the most prominent Chinese mathematicians, Zu Gengzhi 祖 　 之, or Zu Geng 祖 　, is referred to in the book as Zu Kengzhi, except the caption on p. 47 where

collection of mathematical treatises used for mathematical education in the mathematical college of the Imperial University of the Sui (581-618) and Tang (618-907) dynasties, and outlines the establishment of state mathematics education in Japan. Chapter 4, "The mathematics under the Song and Yuan [dynasties]," presents the socio-cultural context in which Chinese mathematics reached its zenith. In particular, Yabuuchi focuses on the philosophical teachings and social networks of Neo-Confucianism and Daoism which, he argues, stimulated the development of certain branches of mathematics. A large part of this chapter (pp. 66-84) is devoted to the methods of solution of algebraical equations of higher degrees involving the original Chinese notation for polynomials, and includes brief yet substantial surveys of the most prominent mathematicians of the period. The chapter ends with a discussion of scientific contacts between China and Islamic countries. Chapter 5, "The development of popular mathematics," is devoted to the methods of counting with the abacus (apparently used in China since the Song dynasty) and to two grand masters of the Ming dynasty, Wu Jing 吳敬 (ca. 1450) and Cheng Dawei 程大位 (1533-1592). Chapter 6, "The introduction of European mathematics," focuses primarily on the activity of the Jesuits in the late Ming to early Qing dynasties, in particular, on the introduction of Euclidean geometry, logarithms and trigonometry. Here the reader will also find a brief presentation of the activities of the most outstanding mathematicians of that time, Mei Wending 梅文鼎 (1633-1721) and Ming Antu 明安圖 (?-1764). Chapter 7, "Renewal of traditional mathematics," is devoted to the research of the late 18th and 19th century scholars on traditional Chinese mathematics; it features two outstanding *literati*, Dai Zhen 戴 震 (1724-1777) and Ruan Yuan 阮 元 (1764-1849). Chapter 8, entitled "Epilogue," focusses on the overwhelming introduction of modern Western science that followed the Opium Wars and the opening of the

---

his first name is transliterated as "Gengzhi." As for other authors, the transliteration "Kengzhi" was used by Jean-Claude Martzloff in his *Histoire des mathématiques chinoises*. Paris etc.: Masson, 1987, while in its English version (*A History of Chinese Mathematics*. Berlin/Heidelberg: Springer, 1997) Zu Gengzhi is systematically mentioned by Martzloff as Zu *Xuan*. Moreover, in English publications using Wade-Giles system of transliteration the name is usually spelled as Tsu Keng or Tsu Keng-chih. Some dictionaries provide for the character two readings *xuan* and *geng* (in *pinyin* transliteration system), e.g. *Zhongwen dacidian* (Taibei: Zhongguo wenhua yanjiusuo, 1962), vol. 16, p. 81, no. 14364, or *Kangxi zidian* (Hong Kong: Zhonghua shuju, 1995), p. 425, while the *Hanyu dacidian* (Shanghai, 1990), vol. 5, p. 804, gives only the reading *xuan*, and lists the name "祖 " among the examples of the use of this character in personal names with the reading *xuan*. I was unable to find the reading *keng* for the character in the dictionaries available to me; most probably it is the Wade-Giles transliteration (equivalent to pinyin transliteration *geng*) mistaken for a pinyin transliteration. I am afraid that without a special footnote those readers of Yabuuchi's book who do not read Chinese will hardly be able to identify the seemingly different individuals referred to in various recent Western publications as Zu Kengzhi, Zu Gengzhi, Zu Geng, Zu Xuan, Tsu Keng, and Tsu Keng-chih.

country in the late 19th century; here the outlines of the lives of Li Shanlan 李善蘭 (1811-1882) and Hua Hengfang 華蘅芳 (1833-1902), two leading mathematicians of the period, are provided. Yabuuchi concludes his account with paragraphs devoted to a comparison of the introduction of Western mathematics into China and Japan. The indices list the personal names, technical terms, and the original titles in Oriental languages (referred to in the body of the book with their French translations).

The only regretful aspect of the book is the complete lack of references. This can be partly explained by the fact that the original Japanese version was published in a series of popularising books addressed to a large audience of non-specialists. Similarly, in France the book was published in a series of *vulgarisation*, and this, probably, was the reason why the translators did not add even a short selected bibliography of recent publications in Western languages that certainly might have been of great help for those who would like to deepen their acquaintance with the subject.

However, to evaluate adequately a book originally published more than 25 years ago and thus seemingly outdated, the reader should be aware that there exist only a few books devoted to the subject in European languages. Among them are the monographs of E. I. Berezkina,[3] Li Yan and Du Shiran,[4] and Jean-Claude Martzloff.[5] Each of them is not free from shortcomings, which can be briefly outlined as follows.[6] Apart from the fact that Berezkina's book is written in Russian and thus is available to only a limited number of Western readers, it contains numerous controversial points promptly noticed by A. P. Yushkevich soon after its publication.[7] The translation of the book of Li Yan and Du Shiran

---

3 El'vira Ivanovna Berezkina, *Matematika drevnego Kitaya* (Mathematics of ancient China), Moscow: Nauka, 1980.

4 Li Yan and Du Shiran, *Chinese Mathematics: A Concise History*, transl. by J. N. Crossley and Anthony W.-C. Lun, Oxford: Clarendon Press, 1987.

5 Jean-Claude Martzloff, *Histoire des mathématiques chinoises*, préfaces de J. Gernet et J. Dhombres, Paris etc.: Masson, 1987/1988. English edition (with considerable emendations by the author): *A History of Chinese Mathematics*, with forewords by Jaques [sic] Gernet and Jean Dhombres, transl. by Stephen S. Wilson, Berlin/Heidelberg: Springer, 1997.

6 I do not discuss here three other outstanding accounts, Mikami Yoshio's *The Development of Mathematics in China and Japan*, Leipzig 1913 (repr. 1961, 1974 by Chelsea), Wang Ling and Joseph Needham's mathematical chapter of Needham's *Science and Civilisation in China* (Cambridge: Cambridge University Press) published in Volume III of the series in 1959, and Adol'f P. Yushkevich's *Istoriya matematiki v srednie veka* (History of mathematics in the Middle Ages), Moscow: GIFML, 1961, later translated into several languages. Each of these works was pioneering in its time.

7 Adol'f P[avlovich] Yushkevich, "Issledovaniya po istorii matematiki v drevnem Kitae" (Research on the history of ancient Chinese mathematics). *Voprosy istorii estestvoznaniya i tekhniki* (Current issues in the history of natural sciences and technology) no. 3 (1983), pp. 125-136.

contains numerous unreliable translations of the quotations from mathematical and philosophical texts written in classical Chinese and left in the original version without translation;[8] moreover, it appears that the editorial work for this book was not done by professionals, as suggested by the mere fact that the figure portraying Fu Xi 伏 犧 and Nü Wa 女 媧 on the second page of its Oxford edition was printed upside-down. Both editions of Martzloff's book, enthusiastically assessed by numerous reviewers, also contain a number of shortcomings that the author of these lines hopes to discuss elsewhere, given the limited scope and volume of the present article.

Yabuuchi's book, despite its age, its conciseness, its strong emphasis on popularisation, the lack of bibliography, as well as the lack of updates that one might have expected to be done by the translators, will certainly occupy a prominent position in this sequence of works, and from many viewpoints can be considered a state-of-art masterpiece. There are several reasons for this high evaluation. Firstly, even though the author worked mainly in the field of traditional Chinese astronomy, he was thoroughtly acquainted with the original mathematical texts. Moreover, he found the right way to present them to the modern reader. From the very first pages devoted to the ancient methods of representing numbers to the very end of the book where the Chinese mathematicians of the early 20th century are discussed, the author does not change his pace, telling his story in a well-structured, clear, sober, and energetic way. Secondly, the book provides a synthesis of two approaches to the phenomenon of Chinese mathematics: while other authors have tried to divide their historical accounts into "social" and "cognitive" parts,[9] Yabuuchi succeeded in writing a history which organically combines together these two complementary components. Last, but not least, the book is nicely illustrated, a considerable contribution to its success. As one can judge on the basis of the list of photographic credits (p. 192), the translators added several dozen photographs, pictures, and diagrams to the original text, while numerous boxes provide clear geometrical diagrams and detailed explanations of mathematical

---

[8] For example, one of the famous phrases from the *Dao de jing* reads *shan shu bu yong chou ce* 善 數 不 用 籌 策, "a skilful calculator does not use [counting] tallies and strips" (Chap. 27). The translation found in the English edition of Li Yan and Du Shiran (p. 7) is: "Those well-versed in calculation use neither counting rods nor texts" (sic). (It remained unspecified whether the translators meant mathematical texts or texts in general.)

[9] For example, Martzloff's book (1997) is divided into two parts entitled "The context of Chinese mathematics" and "The content of Chinese mathematics." Elements of the same approach can be found, for example, in Needham (1959) where the "Survey of the principal landmarks in Chinese mathematical literature" (pp. 18-53) as well as the sections on the transmission (pp. 146-150) and on the comparison of mathematics in China and the West (pp. 150-168) are separated from the sections devoted to the history of particular mathematical methods.

methods. Together with the tastefully chosen picture for the cover, this has endowed the books with a refined esthetical dimension.

One cannot expect, however, a review paper to avoid saying at least a few words about the shortcomings of the book under consideration. Besides the above-mentioned regrettable lack of a selected bibliography, only small and almost insignificant shortcomings can be reported. On p. 35 the author states that in Chapter 8 of the *Computational Procedures of Nine Categories* (*Jiuzhang suanshu*), the simultaneous linear equations in 2 and 3 unknowns were solved, while the chapter also contains problems involving the simultaneous equations in 4 and 5 unknowns;[10] the same statement is reiterated on p. 67. On p. 40, the author states that the Chinese were calculating the volumes of solid figures in "evaluating the number of small cubes called *qi* [that constituted the solids]," whereas the term *qi* 棋 was used by Liu Hui to designate the solids of several "basic" types that included, besides cubes, also prisms and pyramids of particular shape.[11] Moreover, Yabuuchi's interpretation of Liu Hui's infinitesimal procedure of the calculation of the volume of a pyramid (p. 47) is incomplete.[12] On p. 45, the author states that Liu Hui (fl. ca. 263) evaluated the area of a circle with the help of inscribed and circumscribed regular polygons, which is not correct: to evaluate the area, Liu Hui used sequences of inscribed polygons and of circumscribed rectilinear figures of special kind.[13] In some cases, it remains unclear to the reader that Yabuuchi's statements are mere hypotheses rather than well-established facts, yet the translators do not comment on them (short footnotes might have been very useful). This is the case, for example, with the approximate value 3.1416 for pi credited by some authors to Liu Hui, and by others to Zu Chongzhi; Yabuuchi adopts the former hypothesis without mentioning the latter (p. 46). The method of computation of this value presented on the same page is simply a reconstruction adopted by certain Chinese scholars that still awaits proof.[14] On p. 124, Yabuuchi suggests that the term *jihe* 幾何 found in the title of the Chinese translation of *Euclid's Elements* by Matteo Ricci and Xu Guangqi is a phonetical transcription of the radical "geo-" in the word

---

[10] Qian Baocong 錢寶琮 (ed.), *Suanjing shishu* 算經十書 (Ten Computational Treatises; Beijing: Zhonghua shuju, 1963, 2vols.), vol. 1, pp. 232-236. For a thorough study of the simultaneous linear equations in the *Jiuzhang suanshu*, see Ma Li, *Studies of the Chinese Rectangular Array Algorithm in Nine Chapters* (Göteborg: Chalmers University & Göteborg University, 1994).

[11] For more details, see Donald B. Wagner, "An Early Derivation of the Volume of a Pyramid: Liu Hui, Third Century AD," *Historia Mathematica* vol. 6 (1979), pp. 164-188.

[12] A more adequate interpretation can be found in the above-mentioned paper of D.B. Wagner.

[13] See Lam Lay-Yong and Ang Tian-Se, "Circle measurements in Ancient China," *Historia Mathematica* vol. 13, no. 4 (1986), pp. 325-340; Alexeï Volkov, "Calculation of pi in Ancient China: From Liu Hui to Zu Chongzhi," *Historia Scientiarum* vol. 4, no. 2 (1994), pp. 139-157.

[14] For more details, see Volkov (1994).

"geometry," a theory questioned by several scholars.[15] For more general, yet no less controversial statements, see p. 79, where the author discusses the algebraic equations $ax^2 + bx + c = 0$ studied by the Chinese. Since the coefficients *a*, *b*, and *c* were always represented with concrete numbers, Yabuuchi concludes that "Here we have an inevitable consequence of the fact that Chinese mathematics was strongly related to the practical." However, the fact that the algebraic equations were always introduced with the numerical coefficients can hardly ensure that traditional Chinese mathematics was purely "pragmatic." On p. 53 Yabuuchi mentions the problem of the "calculation" of the sex of an unborn child found in the *Sunzi suanjing* 孫子算經 (Computational treatise of Master Sun) and in the Japanese *Kuchizusami* 口遊 (preface 970); he states that the computation involves the month in which the baby is expected to be born, while the texts actually do not specify whether the date is that of expected birth or of conception.[16] In a few cases the translators did not provided footnotes that would be indispensable; for example, the book mentions "the Emperor Shizu of the Yuan [dynasty]" (p. 68), but the identity of this emperor (missing in the Index), who was rather better known as Kubilai Khan (r. 1260-1295) (mentioned on pp. 88 and 97), remains undisclosed.

There are only a few cases where one could suggest slight modifications of the translation, otherwise brilliantly done by C. Jami and K. Baba. On p. 50, it is stated that Zhen Luan 甄鸞 (fl. ca. 560) was "the compiler of two sections of the latter [book, that is, of the *Wujing suanshu* 五經算術 (Computational Methods in Five Confucian Classics)],"[17] while Yabuuchi, most probably, suggested that Zhen Luan authored two books, namely, the *Wucao suanjing* 五曹算經 (Com-

---

[15] See, for example, Peter M. Engelfriet, *Euclid in China* (Leiden etc.: Brill, 1998), pp. 138-141.

[16] For the relevant excerpt from the *Sunzi suanjing* 孫子算經 see Li Yan, "Zhong suan shuru Riben de jingguo" 中算輸入日本的經過 (The process of transmission of Chinese mathematics to Japan), in Li Yan, *Zhong suan shi luncong* 中算史論叢, Beijing: Kexue, 1955, vol. 5, pp. 168-186 (originally published in 1925), p. 174, and an emended version in Qian Baocong (ed*.), Suanjing shishu*, vol. 2, p. 322; for a translation, see Lam Lay Yong and Ang Tian Se, *Fleeting footsteps*, Singapore: World Scientific, 1992, p. 182 (see also pp. 122-123). For the Japanese version of the problem, see Minamoto no Tamenori 源爲憲, *Kuchizusami* 口遊, in *Zoku gunsho ruiji* 續群書類從, Tokyo: Heibunsha, 1982, vol. 64 (32a), pp. 61-85; for the discussed excerpt see p. 84; the quotations from the Japanese treatise found in Mikami Yoshio 三上義夫, "九九に就きて" (On the 'Nine Nines'), *Tôyô gakuhô* vol. 11 (1921), pp. 102-118 on p. 108, and in Li Yan, *ibid*., differ slightly. Yabuuchi does not mention that a very similar problem is found in the *Jiuzhang suanfa bilei daquan* 九章算法比類大全 (The Great Summa of the Generic [Methods related to] the Computational Procedures of Nine Categories) by Wu Jing 吳敬 (fl. 1450) and in the *Suanfa tongzong* 算法統宗 (Systematical Treatise on Counting Procedures) by Cheng Dawei 程大位(1533-1592).

[17] This statement sounds even more strange given that the extant version of the *Wujing suanshu* contains only two sections (*juan*); see Qian Baocong (ed.), *Suanjing shishu*, vol. 2, pp. 441-484.

putational Treatise of Five Departments) and the *Wujing suanshu* 五經算術.[18] On p. 82, the reader finds a mention of the "numbers of grand expansion" (in plural) mentioned in the *Book of Changes* (*Yi jing* 易經), while the *Xici zhuan* 繫辭傳, the Great Commentary of the *Book of Changes* where this concept was originally introduced, specified only one "number of grand expansion" (*da yan zhi shu* 大衍之數), 50.[19] On p. 158, the reader is told that "the goal of [ancient Chinese] astronomy [was] an elucidation of the laws of nature...," and it could make the reader believe that the notion of the laws of nature existed in traditional China; most probably, the original Japanese text contains an expression that might have been translated slightly differently to avoid the controversy.

However, all these minor critical remarks cannot diminish the high value of the book of the late Professor Yabuuchi Kiyoshi. One can hope that his other works, as well as the works of other outstanding Chinese and Japanese historians of science of the 20th century, will become equally available to a general Western audience soon.

---

[18] The latter book was certainly authored by Zhen Luan 甄鸞, given the numerous occurrences of the phrase "Zhen Luan explains" (甄鸞按) found in the text. As for the former, Zhen Luan's authorship of it was suggested in the bibliographical chapters of the dynastic histories (*Jiu*) *Tangshu* (舊) 唐書 and *Xin Tangshu* 新唐書.

[19] *Xici zhuan*, I, 8.